Numerical and Experimental Study of a Virtual Quadrupedal Walking Robot - SemiQuad

Yannick Aoustin, Christine Chevallereau, Alexander Formal'Skii

Numerical and Experimental Study of a Virtual Quadrupedal Walking Robot - SemiQuad *

Yannick Aoustin†, Christine Chevallereau† and Alexander Formal'sky‡

†*Institut de Recherche en Communications et Cybernétique de Nantes U.M.R. 6597 1 rue de la Noë, BP 92101 F-44321 Nantes cedex 3, France. Fax: 33 2 40 37 69 30, Tel: 33 2 40 37 69 48 E-mail: Yannick.Aoustin@irccyn.ec-nantes.fr; Christine.Chevallereau@irccyn.ec-nantes.fr*

‡*Institute of Mechanics, Moscow Lomonosov State University, 1, Mitchurinski Prospect, Moscow 119192 Russia. Fax: 7 095 939 32 74, Tel: 7 095 939 26 28 E-mail: formal@imec.msu.ru*

Abstract. *SemiQuad* is a prototyped walking robot with a platform and two double-link legs. Thus, it is a five-link mechanism. The front leg models identical motions of two quadruped's front legs, the back leg models identical motions of two quadruped's back legs. The legs have passive (uncontrolled) feet that extend in the frontal plane. Due to this the robot is stable in the frontal plane. This robot can be viewed as a "virtual" quadruped. Four DC motors drive the mechanism. Its control system comprises a computer, hardware servo-systems and power amplifiers. The locomotion of the prototype is planar curvet gait. In the double support our prototype is statically stable and over actuated. In the single support it is unstable and under actuated system. There is no flight phase. We describe here the scheme of the mechanism, the characteristics of the drives and the control strategy. The dynamic model of the planar walking is recalled for the double, single support phases and for the impact instant. An intuitive control strategy is detailed. The designed control strategy overcomes the difficulties appeared due to unstable and under actuated motion in the single support.

Due to the control algorithm the walking regime consists of the alternating different phases. The sequence of these phases is the following. A double support phase begins. A fast bend and unbend of the front leg allows a lift-off of the front leg. During the single support on the back leg the distance between the two leg tips increases. Then an impact occurs and a new double support phase begins. A fast bend and unbend of the back leg allows the lift-off of the back leg. During the single support on the front leg the distance between the two leg tips decreases to form a cyclic walking gait.

The experiments give results that are close to those of the simulation.

Keywords: virtual quadruped, dynamical model, passive impact, dynamically stable gait, intuitive control law, simulation, experiment.

1. Introduction

It is possible to distinguish different families of walking robots. Firstly, the statically stable robots, their motion is such that the projection of the mass center is always inside the support polygon i.e, polygon formed by the stance feet, see for example (Gurfinkel et al, 1981; Kaneko et al, 1985; Klein and Briggs, 2000; Hirose and Kato, 2000; Saranli et al, 2001; Conte et al, 2003; Estremera et al, 2003). Usually, only slow motions are achieved in the static stability mode, see (Ting et al, 1994). For high speeds, recently built machines with compliant legs do not belong to this statically stable robot category although a support triangle is present, see (Cham and al, 2000; Altendorfer et al, 2001). Secondly, the semi dynamically stable robots. Their motion is such that the Zero Moment Point (ZMP) is inside the support surface (Vukobratovic et al, 1990; Hirai et al, 1998). The feet have a surface contact with the ground. The last family is composed of bipeds, or quadrupeds walking robots, which have sometime less than three point feet on the ground. These walking robots are under actuated mechanisms for some phases of the motion. A free rotation of the robot around the contact point or around the axe defined by the two contact points is possible. The problem of the control law design is more difficult for this family of robots. Many papers are devoted to this kind of walking robots, for example to the bipeds, which are adapted to the human environment, see for example (Aoustin and Formal'sky, 2003; Chevallereau et al,

* Funding for *SemiQuad* was primarily provided by the CNRS and the region Pays de La Loire.

2003; Formal'sky, 1982; Formal'sky, 1997; Miura and Shimoyama, 1984). However, the quadrupeds are also interesting objects of study (Hirose and Yoneda, 1993). In the papers (Fukuoka et al, 2003; Furusho et al, 1995; Hong et al, 1999; Kimura et al, 1999; De Lasa and Buehler, 2001; Poulakakis et al, 2003; Talebi et al, 2000) a dynamically stable locomotion is designed for a quadruped. There are several possible kinds of gait for a quadruped, for example, amble, trot, curvet (Sutherland and Raibert, 1983; Perrin, 1999; Formal'sky et al, 2000; Muraro et al, 2003). Usually two following half steps of a quadruped are not symmetric in the sagittal plane. In the papers (De Lasa and Buehler, 2001; Poulakakis et al, 2003; Talebi et al, 2000) quadrupeds with four telescopic legs are described. The robots use a "walking bound" gait (without flight phase) as our quadruped. Unlike our robot, each of these robot's legs has one actuated rotational hip joint and a passive compliant prismatic joint. With the proposed control strategy, the robot exhibits stable walking. Our Semiquad has knees and each leg has two actuators. Our control strategy is based on a use of the motion of the knee to bend and unbend the leg in order to obtain a taking off of the leg. The forward speed of our quadruped is lower than the speed used by Scout II (De Lasa and Buehler, 2001). But it is difficult to compare the performance of the two control laws because they are used for quadrupeds with quite different design, size, drives. In (Krupp, 2000) the prototype that is considered is constrained to motion in a plane. It has a platform, two hip joints and two knee joints. Each joint is driven by a force controlled Series Elastic Actuator, see (Pratt et Williamson, 1995). The design of this robot is mainly devoted to the running gait.

The dynamically walking quadrupeds present an interesting challenge. Therefore we decided to build a prototype, which is a semi quadruped or virtual quadruped. It has two legs, which are linked by a platform. In the sagittal plane with a walking curvet gait it can be viewed as a quadruped robot using the virtual leg principle (Sutherland and Raibert, 1983), if the motions of the front legs (respectively the motions of the back legs) are synchronized. Our prototype walks in dynamically stable mode.

The article is organized as follows: Section 2 describes our virtual quadruped mechanism. A dynamical model of the mechanism is defined in Section 3. The control strategy is explained in Section 4. Section 5 is devoted to the simulation of *SemiQuad*. Section 6 presents the first experimental results. Section 7 contains our conclusion.

2. Description of the Mechanism

We consider a curvet gait of a quadruped when both front legs move identically with respect to the body and both back legs move identically as well. In other words, the angles between the body and the thighs of the two front legs (back legs) are always identical. The angles in the knee joints are always identical too. This means that the front legs of our quadruped are coupled and the back legs are coupled as well, see Figure 1 (a). The model of the quadruped with coupled front and coupled back legs, is equivalent (Sutherland and Raibert, 1983) to the model of a quadruped with two virtual legs, see Figure 1 (b). Similar gaits are presented in (De Lasa and Buehler, 2001; Poulakakis et al, 2003; Talebi et al, 2000; Krupp, 2000). Thus, we study a virtual quadruped with two legs - *SemiQuad*. Photo of our *SemiQuad* is presented in Figure 2.

Figure 1. (a) Quadruped with coupled front and coupled back legs, (b) Quadruped with two virtual legs.

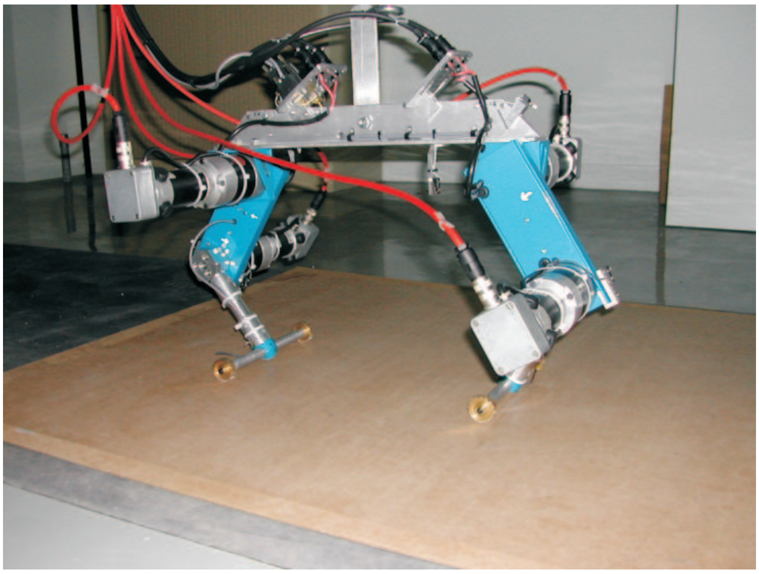


Figure 2. Photo of *SemiQuad*.

The prototype is composed of a platform and two identical double-link legs with knees. The legs have passive (uncontrolled) feet that extend in the frontal plane. Thus, the mechanism can execute planar walking only. So, only 2-D motion in the sagittal plane is considered. There are four electrical DC motors with gearbox reducers to actuate the haunch joints between the platform and the thighs and the knee joints. Using a dynamic simulation, we have chosen parameters of the prototype (the sizes, masses, inertia moments of the links) and convenient actuators. The parameters of the four actuators with their gearbox reducers are specified in Table I. The lengths, masses and inertia moments of each link of *SemiQuad* are specified in Tables II with the help of Figure 3.

Table I. Parameters of actuators

DC motor + gearbox	*length (m)*	*mass (kg)*	*gearbox ratio*	*rotor inertia* $(kg{\cdot}m^2)$	*Electromagnetical constant torque* $(N\cdot m)/A$
in haunch	0.23	2.82	50	$3.25 \cdot 10^{-5}$	0.114
in knee	0.23	2.82	50	$2.26 \cdot 10^{-5}$	0.086

Table II. Parameters of *SemiQuad*

physical parameters of each link	*mass (kg)*	*length (m)*	*center of mass locations (m)*	*moment of inertia* $(kg{\cdot}m^2)$ *around the center of mass,* C_i $(i = 1, ..., 5)$
links 1 and 5: shin	$m_1 = m_5 = 0.40$	$l_1 = l_5 = 0.15$	$s_1 = s_5 = 0.083$	$I_1 = I_5 = 0.0034$
link 3: platform + actuators in each haunch	$m_3 = 6.618$	$l_3 = 0.375$	$s_3 = 0.1875$	$I_3 = 0.3157$
links 2 and 4: thigh + actuator in knee	$m_2 = m_4 = 3.21$	$l_2 = l_4 = 0.15$	$s_2 = s_4 = 0.139$	$I_2 = I_4 = 0.0043$

The maximal value of the torque in the output shaft of each motor gearbox is 40 $N{\cdot}m$. This saturation on the torques is taken into account to design the control law in simulation and in experiments. The total mass of the quadruped is 14 kg approximately. The four actuated joints of the robot are each equipped with one encoder to measure the angular position. The angular velocities are calculated, using the angular positions. Currently the absolute platform orientation angle α (see Figure 3) is not measured. The proposed intuitive control does not require this measurement. Each encoder has 2000 pts/rev and is attached directly to the motor shaft. The speed of response or the bandwidth of each joint of the robot is determined by the transfer function of the mechanical power train (motors, gearboxes) and the motor amplifiers that drive each motor. In the case of *SemiQuad*, we have approximately a 16 Hz bandwidth in the mechanical portion of the joints and approximately 1.7 kHz for the amplifiers.

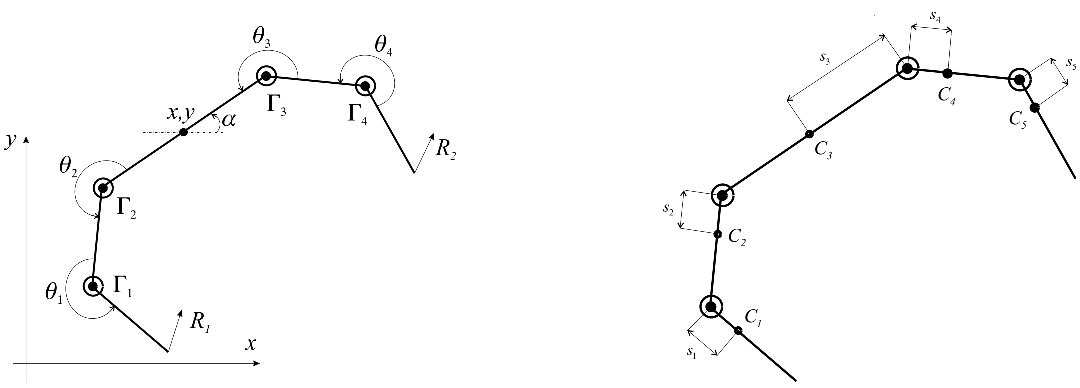
To get the measurement data and to implement the control algorithm, a dSPACE system was selected as real-time control platform. The low-level computation, digital-to-analog and analog-to-digital conversion, as well as the user interface are provided by the dSPACE package. The control computations are performed with a sample period of 5 ms (200 Hz). The software is developed in C language.

3. Dynamic Model

The models for single support and double support motions are presented in this section. The difficulty of the study of the walking gait is that the structure of its dynamical model changes during the walking gait because it consists of a single support phase, a double support phase and an impact.

3.1. Single Support Phase

The SemiQuad's diagram is shown in Figure 3.



(a) (b)

Figure 3. SemiQuad's diagram: Generalized Coordinates, Torques, Forces Applied to the Leg Tips, Locations of mass centers .

The two Cartesian coordinates x, y of the platform mass center and the five orientation angles $(\alpha, \theta_1, \theta_2, \theta_3, \theta_4)^T = q$ of the platform and the legs define vector X of the generalized coordinates. The vector $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)^T$ with four components denotes the applied torques in the haunch and knee joints. Let $R_j = (R_{jx}, R_{jy})^T$ be the force applied to the foot j. When leg j is on the ground, then R_j is the ground reaction. If leg j is the swing leg, then $R_j = 0$. Using the second Lagrange method, motion equations of the *SemiQuad* in the swing phase are obtained and they have the following well-known form, for $j = 1$ or 2,

$$A(q)\ddot{X} + H(q, \dot{q}) = D\Gamma + D_j(q)R_j. \tag{1}$$

Here $A(q)$ is the symmetric, positive definite 7×7 inertia matrix; $H(q, \dot{q})$ is the 7×1 vector of the centrifugal, Coriolis and gravity forces; D is a 7×4 fixed matrix, consisting of zeros and units; $D_j(q)$ is a 7×2 matrix.

Setting null acceleration condition of the stance leg tip

$$D_j^T(q)\ddot{X} + H_j(q, \dot{q}) = 0 \tag{2}$$

implies that both horizontal coordinate and vertical coordinate of that leg tip do not change, if their initial velocities are null. $H_j(q, \dot{q})$ is a 2×7 matrix. Thus, in the single support the number of degrees of freedom is five, but there are only four torques. This means that *SemiQuad* is an under actuated mechanism in the single support motion. Also it is statically unstable during this motion. Thus, the control problem for SemiQuad is difficult.

3.2. Double Support Phase

In the double support, *SemiQuad* has both legs on the ground. Instead of matrix equations (1) and (2) we have the following matrix equations for this phase,

$$A(q)\ddot{X} + H(q, \dot{q}) = D\Gamma + D_1(q)R_1 + D_2(q)R_2. \tag{3}$$

$$\begin{pmatrix} D_1^T(q) \\ \\ D_2^T(q) \end{pmatrix} \ddot{X} + \begin{pmatrix} H_1(q) \\ \\ H_2(q) \end{pmatrix} = \begin{pmatrix} 0 \\ \\ 0 \end{pmatrix} \tag{4}$$

Matrix equation (3) presents the dynamical model. Matrix equation (4) comes from the condition that both leg tips are motionless during the double support motion.

In the double support, *SemiQuad* has three degrees of freedom and four torques applied in the four joints. In this phase, our walking robot is over actuated. It can be statically stable during double support motion.

3.3. Passive Impact Model

During the *Semiquad*'s gait, the impact occurs at the end of a single support phase, when the swing leg tip touches the ground. The instant of impact is denoted by T. We assume that this impact is passive, absolutely inelastic and that the legs do not slip. Given these conditions, the ground reactions at the instant of an impact can be considered as impulsive forces and defined by Dirac delta-functions $R_j = I_{R_j}\delta(t-T)$ $(j = 1, 2)$. Here $I_{R_j}(I_{R_{jx}}, I_{R_{jy}})$ is the vector of the magnitudes of the impulsive reaction in the leg j, see (Formal'sky, 1982).

Impact equations can be obtained through integration of the matrix motion equation (3) for the infinitesimal time from $T-0$ to $T+0$. The torques provided by the actuators at the joints, Coriolis and gravity forces have finite value, thus they do not influence the impact. Consequently the impact equations can be written in the following matrix form:

$$A(q(T))\left(\dot{X}^{+} - \dot{X}^{-}\right) = D_1(q(T))I_{R_1} + D_2(q(T))I_{R_2} \tag{5}$$

Here $q(T)$ denotes the configuration of the quadruped at instant $t = T$, (this configuration does not change at the instant of the impact), $\dot{X}^{-}$ and $\dot{X}^{+}$ are respectively the velocity vectors just before and just after an inelastic impact. The velocity of the stance leg tip $(j = 1)$ before an impact equals to zero,

$$D_1^T(q(T))\dot{X}^{-} = 0 \tag{6}$$

The swing leg $(j = 2)$ after the impact becomes a stance leg. Therefore, its tip velocity becomes zero after the impact,

$$D_2^T(q(T))\dot{X}^{+} = 0 \tag{7}$$

Generally speaking, two results are possible after the impact, if we assume that there is no slipping of the leg tips. The stance leg lifts off the ground or both legs remain on the ground. In the first case, the vertical component of the velocity of the taking-off leg tip just after the impact must be directed upwards. Also there is no interaction (no friction, no sticking) between the taking-off leg tip and the ground. The ground reaction in this taking-off leg tip must be null. In the second case, the stance leg tip velocity has to be zero just after the impact. The ground produces impulsive reactions (generally, $I_{R_j} \neq 0$, $j = 1, 2$) and the vertical components of the impulsive ground reactions in both legs are directed upwards. For the second case, the passive impact equation (5) must be completed by one matrix equations.

$$D_1(q)^T\dot{X}^{+} = 0 \tag{8}$$

The system (5), (7) and (8) consists of 11 scalar equations and contains 11 unknown variables that form the vectors $\dot{X}^{+}$, I_{R_1} and I_{R_2}. In general, the result of a passive impact is dependant on two factors: the quadruped's configuration at the instant of an impact and the direction of the swing leg tip velocity just before impact (Formal'sky, 1982).

4. Control Strategy

By analogy with the animal walk, the gait studied here consists of alternating phases of single and double support. There is no flight phase. *SemiQuad* must jump with front or back leg to realize this walking gait. It is not possible to adopt another way without leg sliding.

Remark: *It is possible to define a motion of SemiQuad, using only double support phase with sliding of the leg tips along the supporting surface.*

We have developed an intuitive control strategy for *SemiQuad* and tested it first in simulation (Aoustin and Formal'sky, 1998) to study its feasibility and then experimentally. Let us describe this control strategy.

In Figure 4, we show stick configurations to illustrate our control strategy. The projection on the ground of the initial position of the platform center at the beginning of the step is referenced by a *x-mark*.

Let us consider a current half step k, which is composed of a double support and a single support with the stance back leg. The next half step $k+1$ is composed of another double support and a single support on the stance front leg.

The half step k starts from the configuration which is shown in Figure 4a. This configuration is symmetrical with respect to the vertical axis crossing the platform center. The projection of the platform center is in the middle of the leg tips.

At the beginning of the current half step k, *SemiQuad* has to prepare its front leg tip to lift off. Therefore, initially in the double support, *SemiQuad* transfers its mass center backward and lowers its platform (see the configuration in Figure 4b). To perform this motion we compute on line the reference trajectories for three chosen joint angles as polynomials of third order in time with constant values at the end of the Semiquad's motion. The controlled joint angles are both knee joints and the back haunch joint. Both front and back legs are bending their knees during this double support.

Then the front leg sharply unbends (see the configuration in Figure 4c). A large constant torque (in open loop) is applied to the front leg knee joint to unbend it. At this time the front leg pushes the ground with a large force. When the knee joint angle of the front leg reaches the prescribed value we invert this torque (in open loop) and the front leg bends its knee once more. At this instant the front leg tip looses contact with the ground and the single support phase on the stance back leg starts (see the configuration in Figure 4d). When we apply torques in open loop in the front knee, the control law in the joints of the back leg maintains their configuration constant. The torque in the front haunch is equal to zero: this joint is passive.

During the single support, four torques are applied to the joints to track reference trajectories. The reference trajectories for the inter-link angles are such that the distance between the leg tips increases during the single support. Thus the front leg tip moves forward (see the configuration in Figure 4e). The reference trajectory $\theta_{ref,i}(t)$ $(i = 1, ..., 4)$ for each inter-link angle (see Figure 3a) is a polynomial function in time of third order, but before the impact, the desired inter-link angles become constant. Using this strategy, we obtain the desired final configuration of our prototype before the impact even if the duration of the single support phase is not exactly the expected one. Obtaining the final configuration before the impact helps considerably to reduce the error in the quadruped's configuration at the beginning of the next half step $k+1$. The single support phase ends at the instant when the front leg touches (with an impact) the ground (see the configuration in Figure 4f). After the half step k the center of the platform has moved forward. The configuration in Figure 4f is the last configuration of the half step k and simultaneously it is the initial configuration of the half step $k+1$.

Just after the impact of the front leg, the following half step $k+1$ begins. At the beginning of this half step, *SemiQuad* prepares its back leg tip to lift off. Therefore, initially in the double support of the half step $k+1$, *SemiQuad* transfers its mass center forward and lowers its platform (see the configuration in Figure 4g). Similarly to the previous half step k, to perform this motion we compute on line the reference trajectories for three chosen joint angles as polynomials of third order in time with constant parts at the end of the Semiquad's motion. The controlled joint angles are both knee joints and the front haunch joint. Both front and back legs are bending their knees during this phase.

Then the back leg sharply unbends (see the configuration in Figure 4h). A large constant torque (in open loop) is applied to the back leg knee joint to unbend it. At this time the back leg pushes the ground. After we invert this torque (in open loop) and the back leg bends its knee once more. At this instant the back leg tip looses contact with the ground and the new single support phase on the stance front leg starts (see the configuration in Figure 4i). When we apply torques in open loop in the back knee, the control law in the joints of the front leg maintains their configuration constant. The torque in the front haunch is equal to zero: this joint is passive.

During the single support phase on the back leg, the distance between the leg tips decreases (see the configuration in Figure 4j) and becomes equal to the distance which is shown in Figure 4a. Before the impact, the desired inter-link angles become constants. To freeze the configuration of *SemiQuad* before the impact with the ground of of the swing leg tip helps considerably to reduce the error in the quadruped configuration at the beginning of the next half step $k+2$. The single support phase ends at the instant when the back leg touches (with an impact) the ground. After the half step $k+1$ the center of the platform has moved forward. After the double support phase of the half step $k+2$ starts.

Remarks about ways to design the reference trajectories:

Let the initial (current) configuration, the desired terminal configuration, the initial (current) velocity, and the desired duration τ of the transferring from an initial to terminal configuration be given for a phase.

Thus, for each inter-link angle the values $\theta_{ref,i}(0)$, $\theta_{ref,i}(\tau)$, $\dot{\theta}_{ref,i}(0)$, for $(i = 1, ...)$ are known.

To help the transition between phases in presence of perturbation, a fixed interlink configuration is imposed before the end of the single or double support phase. In consequence, the null desired final interlink velocity is chosen: $\dot{\theta}_{ref,i}(\tau) = 0$ $(i = 1, ...)$.

We have four constraints to satisfy for each relative joint, thus three order polynomial function of time can be used:

$$\begin{aligned} t \le \tau, \quad & \theta_{ref,i}(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3 \\ t \ge \tau, \quad & \theta_{ref,i}(t) \equiv \theta_{ref,i}(\tau) \end{aligned} \qquad (i = 1, ...) \tag{9}$$

The last line means that the desired inter-link angles become constants before the impact. Now we can solve a boundary value problem, computing the coefficients of polynomials.

The coefficients a_{i0}, a_{i1}, a_{i2} and a_{i3} are computed on line by using the initial joint angles, the initial joint angular velocities and by defining the terminal joint angles $\theta_{ref,i}(\tau)$ $(i = 1, ...)$ and prescribed time τ. If the order of the polynomial equals three, the solution for the coefficients is unique.

In double support phase, two ways can be used to transfer Semiquad *to its desired position. One way is to compute on line the reference trajectories for three chosen joint angles as polynomials in time (i=1,2,4 or i=1,3,4). The fourth relative joint is not controlled. The torque in the fourth joint is equal to zero. Another way is to compute on line a reference trajectory for the platform mass center and for the platform orientation angle as polynomials in time. Then using geometric relations, the corresponding reference trajectories in position and in velocity for the joint angles can be computed. The first way is used for simplicity*

To track the reference trajectories in simulation and in experiments we apply usual PD-controllers (without feedforward term).

The strategy of control is summarize in table III.

Table III. The control strategy

State	*Phases*	*Action*	*Effect*
Double Support	Phase 1	$\theta_i = \sum_{j=0}^{3} a_j t^j, i = 1,2,4$	The mass center moves backward
	Phase 2	$\theta_i = const, i = 1,2,4$	The robot configuration is fixed
	Phase 3	Γ_4 is controlled, $\Gamma_3 = 0$	Prepare the lift-off of front leg
		$\theta_i = const, i = 1,2$	
Single Support on back leg	Phase 1	$\theta_i = \sum_{j=0}^{3} a_j t^j, i = 1,2,3,4$	Distance between legs increases
	Phase 2	$\theta_i = const, i = 1,2,3,4$	The robot configuration is fixed
Double Support	Phase 1	$\theta_i = \sum_{j=0}^{3} a_j t^j, i = 1,3,4$	The mass center moves forward
	Phase 2	$\theta_i = const, i = 1,3,4$	The robot configuration is fixed
	Phase 3	Γ_1 is controlled, $\Gamma_2 = 0$	Prepare the lift-off of back leg
		$\theta_i = const, i = 3,4$	
Single Support on front leg	Phase 1	$\theta_i = \sum_{j=0}^{3} a_j t^j, i = 1,2,3,4$	Distance between legs decreases
	Phase 2	$\theta_i = const, i = 1,2,3,4$	The robot configuration is fixed

5. Simulation

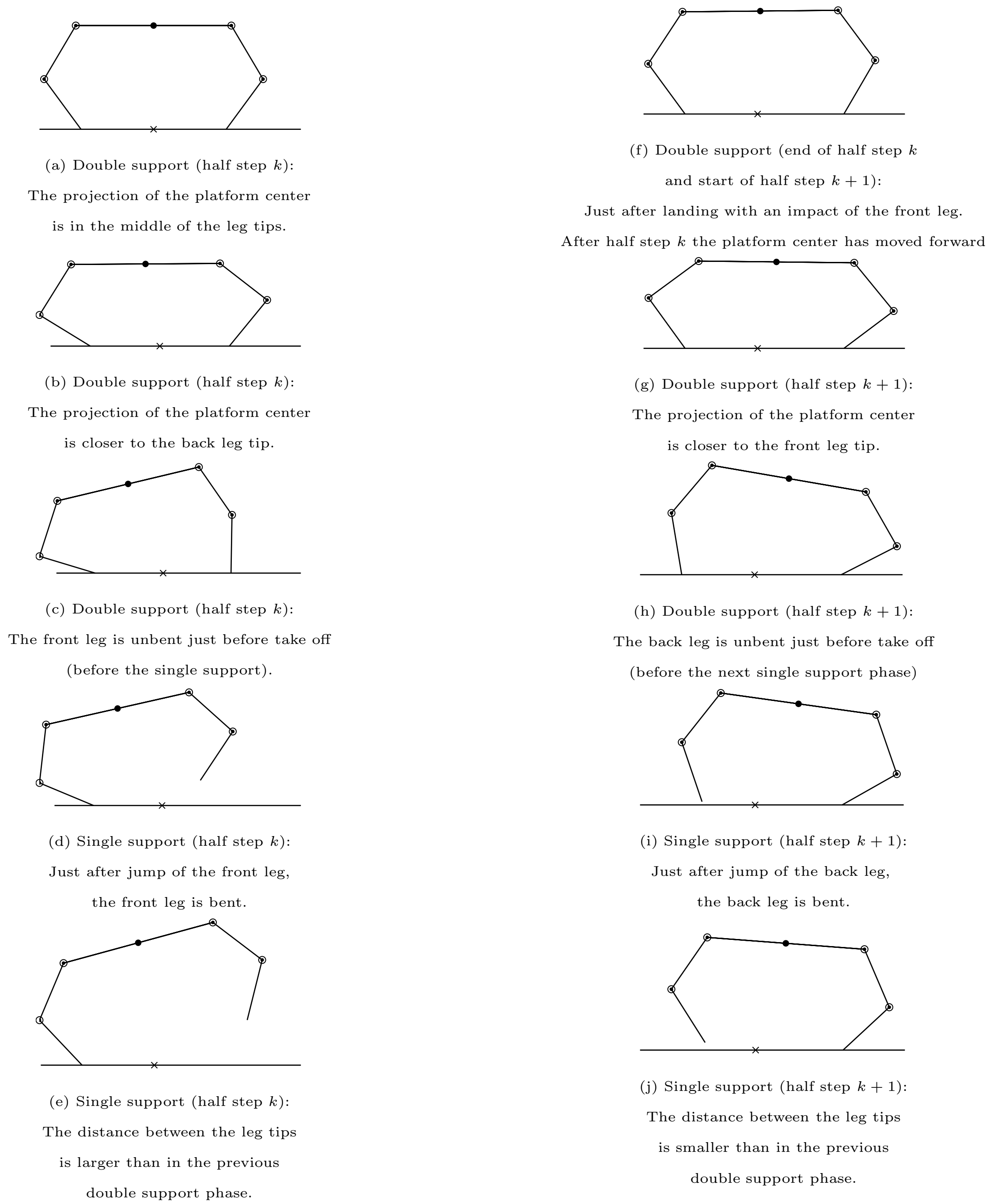
The stick diagram in Figure 4 is derived from the simulation and shows the quadruped motion during two half steps.

Our control law is intuitive. We theoretically do not prove the existence of the cyclic regime. But the simulation shows that the cyclic regime exists and it is stable. In Figure 5, we show the Phase portrait in the plane $(\alpha, \dot{\alpha})$. This portrait for the platform orientation angle α is obtained from the simulation for twenty steps. In Figure 6, the graphs of the functions $\alpha(t)$ and $\dot{\alpha}(t)$ for five steps are shown.

We see from the Figures 5 and 6 angle α changes periodically. There are discontinuities in the function $\dot{\alpha}(t)$ at the impact instants: two discontinuities during two half steps.

In Figure 7, the vertical components of the ground reactions in both legs are shown. These graphs correspond to both half steps k and $k+1$. When the large torque is applied in the front (back) leg knee, the vertical component of the ground reaction in this leg instantaneously increases. When the front (back) leg looses contact the ground reaction in this leg becomes zero and the vertical component of the ground reaction in the back (front) leg instantaneously increases. When the front (back) leg touches the ground the vertical component of the ground reaction in this leg instantaneously increases and the vertical component of the ground reaction in the opposite leg instantaneously decreases.

One objective of the double support phase is to move the mass center to help the lift-off of one leg. In Figure 8, we present the difference between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the back leg tip during the half step k, and the difference between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the front leg tip during the half step $k+1$. During the double support, the distance between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the stance leg tip decreases to reduce the torque around the stance leg tip due to gravity. We can see also from the Figure 8 that initially the absolute value of the torque due to gravity ($Mg{\cdot}0.13\ N{\cdot}m$) is more important than those ($Mg{\cdot}0.096\ N{\cdot}m$) at the beginning of the half step k. This means that SemiQuad has an initial configuration for the half step $k+1$ such that it is more difficult to rotate the mechanism about the front leg tip than to rotate it about the tip of the back stance leg with the initial configuration at the half step k.



(a) Double support (half step k): The projection of the platform center is in the middle of the leg tips.

(b) Double support (half step k): The projection of the platform center is closer to the back leg tip.

(c) Double support (half step k): The front leg is unbent just before take off (before the single support).

(d) Single support (half step k): Just after jump of the front leg, the front leg is bent.

(e) Single support (half step k): The distance between the leg tips is larger than in the previous double support phase.

(f) Double support (end of half step k and start of half step $k+1$): Just after landing with an impact of the front leg. After half step k the platform center has moved forward

(g) Double support (half step $k+1$): The projection of the platform center is closer to the front leg tip.

(h) Double support (half step $k+1$): The back leg is unbent just before take off (before the next single support phase)

(i) Single support (half step $k+1$): Just after jump of the back leg, the back leg is bent.

(j) Single support (half step $k+1$): The distance between the leg tips is smaller than in the previous double support phase.

Figure 4. Plot of half steps k and $k+1$ of *Semiquad* as a sequence of stick figures.

SemiQuad has no actuation at the leg tips. So the gravity has an important effect in the rotation of the robot about the stance leg tip in the single support phase. To evaluate this effect let us consider σ, the angular momentum of the robot about the stance leg tip, which is assumed to act as a pivot (i.e., it does not slip and remains in contact with the ground). The angular momentum balance theorem says that the time derivative of the angular momentum about a fixed point is equal to the sum of the moments of the external forces about that point. Since the motor torques act internally to the robot, their contribution to the momentum balance is zero. The gravity torque is only an external torque, therefore

$$\dot{\sigma} = Mgx_c \tag{10}$$

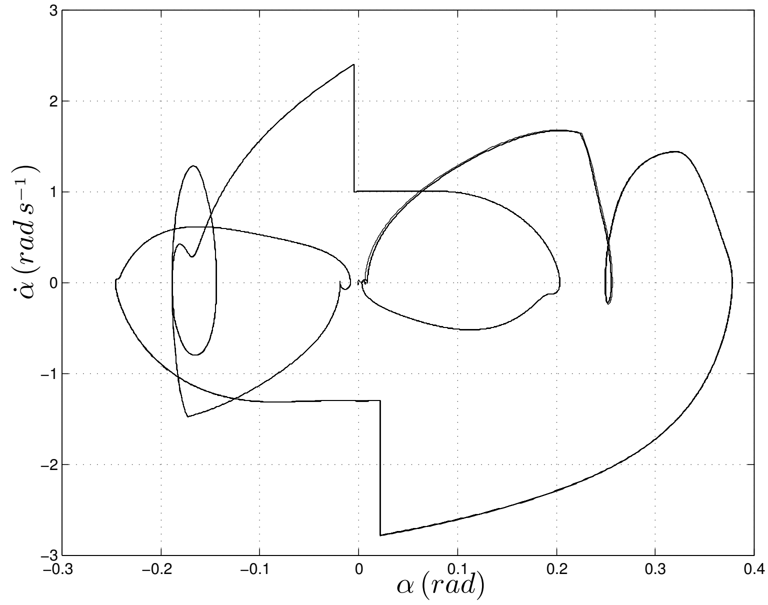


Figure 5. Simulation: Phase portrait in the plane $(\alpha, \dot{\alpha})$.

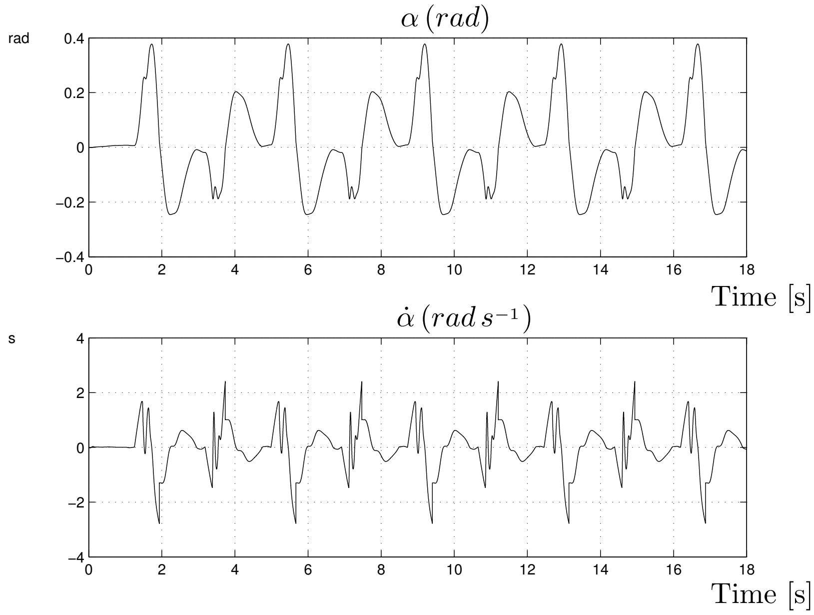


Figure 6. Simulation: Graphs of the functions $\alpha(t)$, $\dot{\alpha}(t)$ for five steps.

Here M is the total mass of SemiQuad, g is the gravity acceleration and x_c is the difference between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the stance leg tip (Figure 8). In Figure 9, the angular momentum around the stance leg tip during the single supports is presented.

During half step k, the angular momentum around the back leg tip decreases strictly monotonically. At the start of the single support the angular momentum is positive, after it becomes negative. During half step $k+1$, the angular momentum around the front leg tip increases strictly monotonically. At the start of the single support the angular momentum is negative, after it becomes positive. The horizontal projection of the mass center (see Figure 8) is always in front (behind) of the tip of the stance back (front) leg during the single support of the half step k $(k+1)$. Thus, contrary to biped walking, during

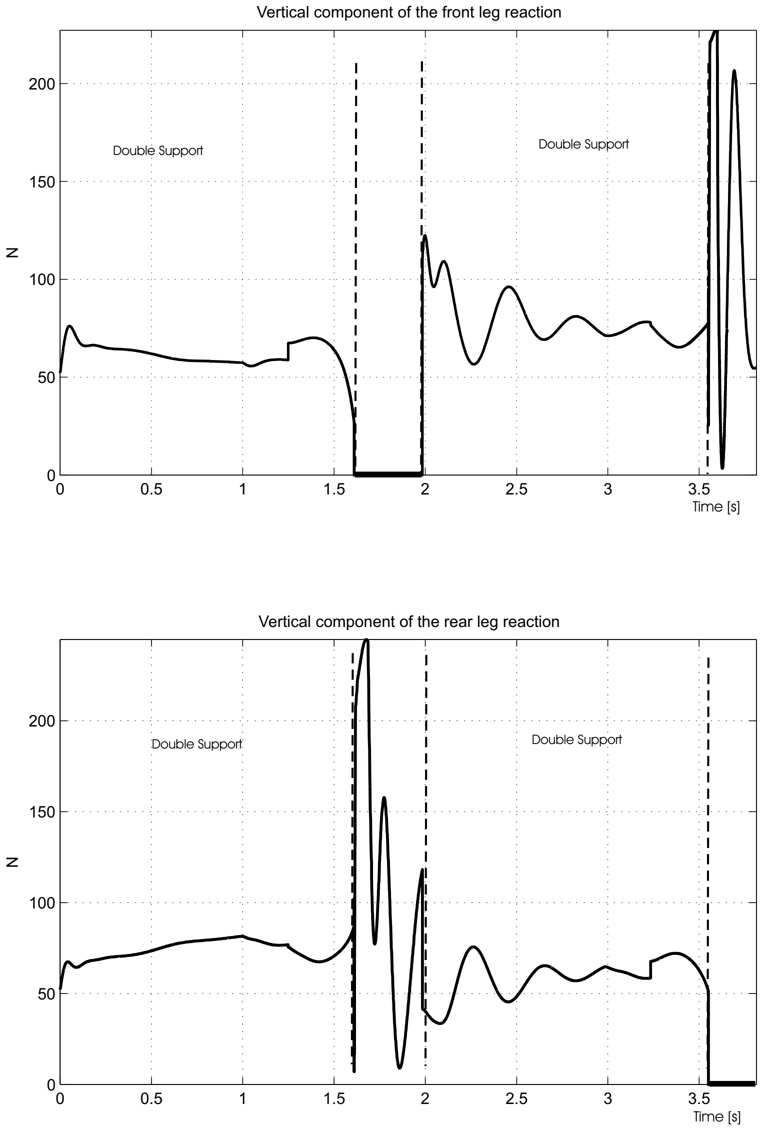


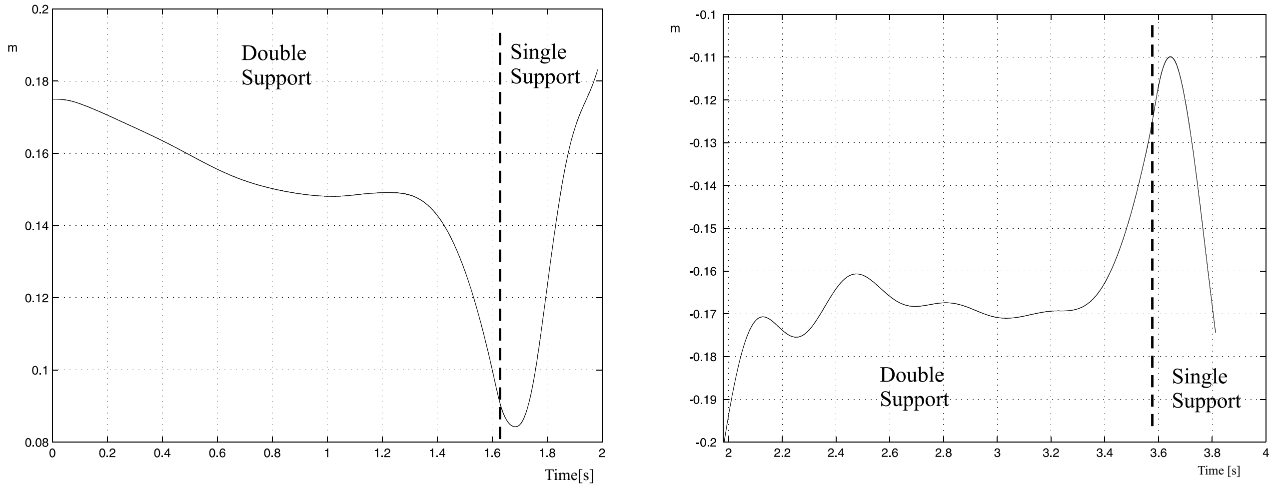
Figure 7. Simulation: Vertical Components of Both Ground Reactions.

both single supports the robot never reaches the unstable equilibrium posture when its mass center is just over the contact point with the ground. SemiQuad rotates backward (forward) about the tip of its back (front) stance leg at the beginning of its motion for single support k $(k+1)$. After the gravity effects inverse this rotation. During the single support, the distance between the x-coordinate of the mass center and the x-coordinate of the stance leg tip decreases initially and after increases. This fact confirms that the gravity effects inverse the rotation of *Semiquad*.

6. Experiments

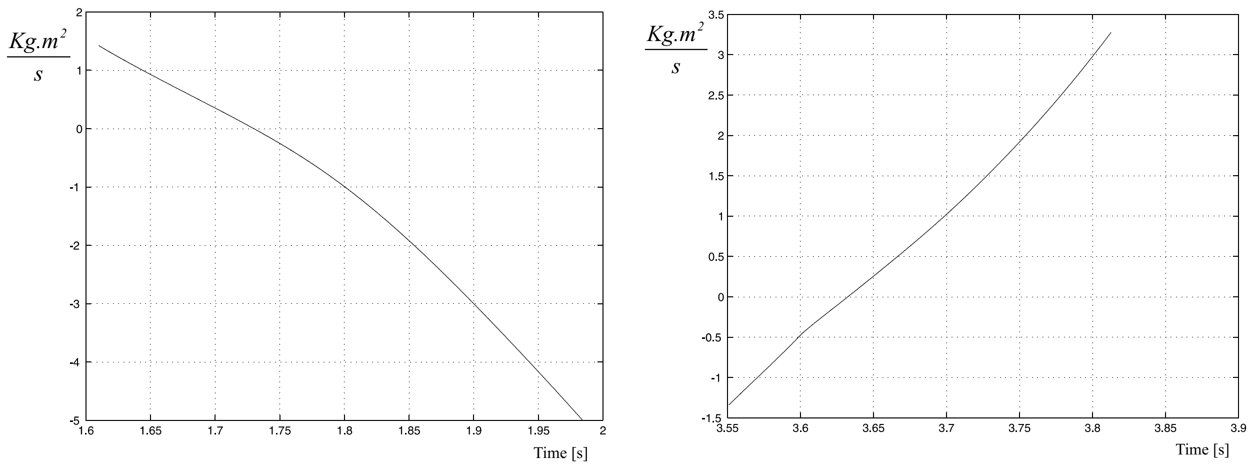
In the experimentations, the designed control algorithm was successfully implemented. Our *SemiQuad* executes more than thirty steps. The information about the absolute orientation of *Semiquad* is not necessary for our intuitive control law. Then currently we do not have a sensor to measure it.

The graphs with the joint angle data of *Semiquad* in simulation and in experimentation are shown in Figure 10. The angle data, desired and measured of the real robot for six steps are shown in Figure 11



(a) (b)

Figure 8. Simulation: a) The difference between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the back leg tip during the half step k, b) the difference between the x-coordinate of the mass center of SemiQuad and the x-coordinate of the front leg tip during the half step $k+1$.



(a) (b)

Figure 9. Simulation: a) Angular momentum about the back leg tip during the single support of the half step k, b)Angular momentum about the front leg tip during the single support of the half step $k+1$.

too. It follows from this Figure 11 that the tracking of the desired angles is very good. There are differences between angles in simulation and experimentation in some instants. These differences would be explained by the fact that no friction effects are considered in simulation. However the forward velocities in simulation and in experiments are closed. During the experiments we evaluate a forward velocity of 12.5 mm/s for *Semiquad*. In simulation this forward velocity is equal to 13.3 mm/s. Furthermore the cost of transport $C = W/(mgx)$, which is the work done by the motors per unit weight and distance of the machine, is almost equivalent for the simulation model, and in experiments. The value of C equals 17947 for the simulation model and 18843 in experiments.

Let us remark that it is possible to increase the forward velocity reducing the duration of the double support.

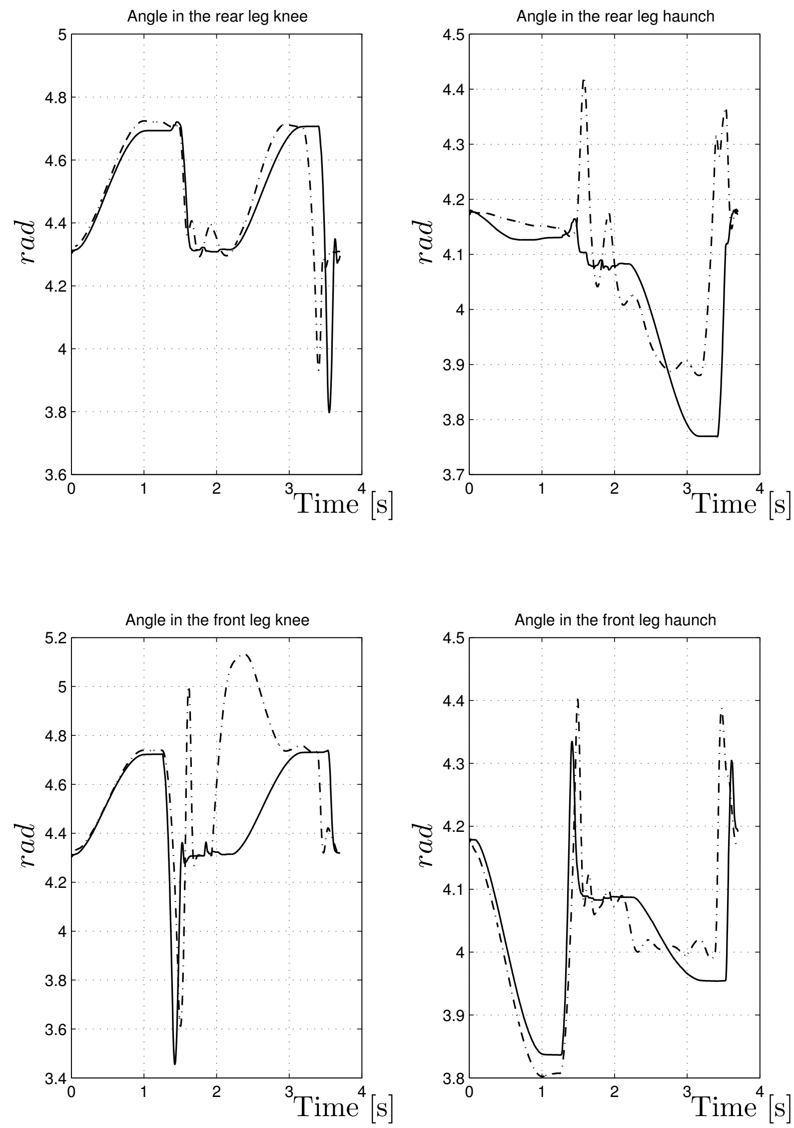


Figure 10. Joint angle data, Experiments (solid), Simulation (dash-dot)

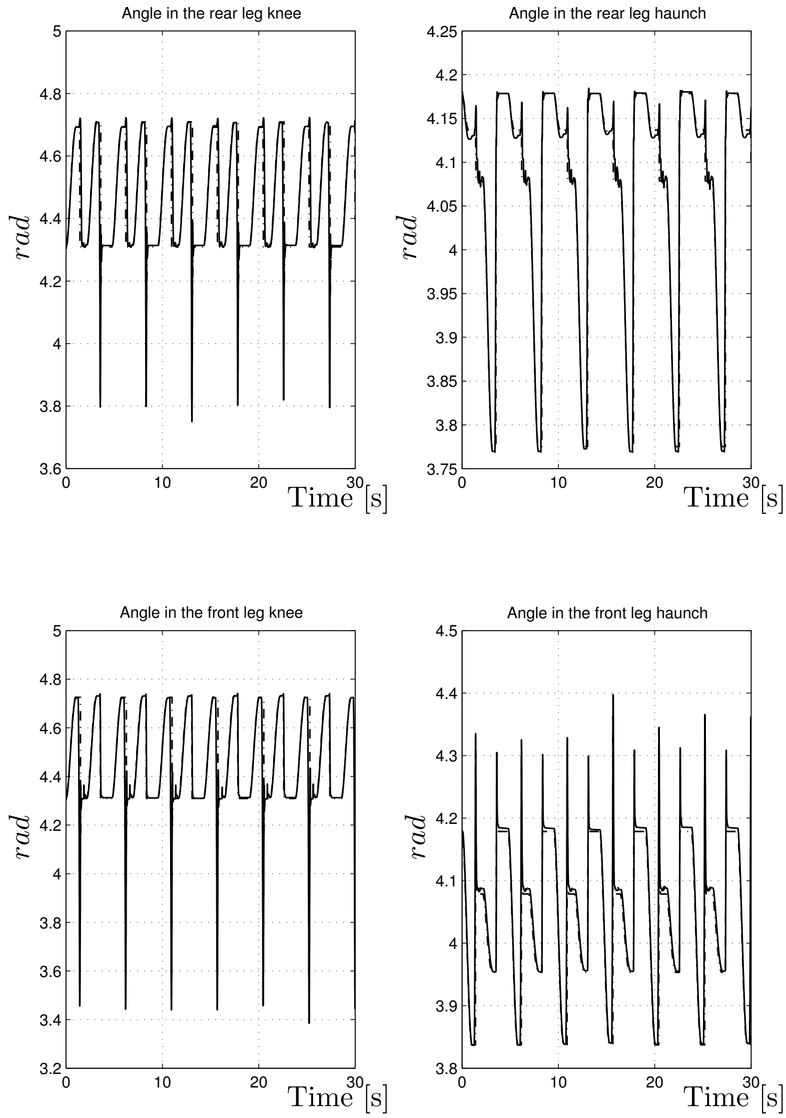


Figure 11. The angle data, Desired (dashed-dot), Actual (solid) of the real robot for six steps

Figure 12 shows a sequence of six pictures of *SemiQuad's* motion during two half steps. *SemiQuad* moves from the left to the right. In Figure 12(a) both legs are on the support surface and the projection of the platform center is in the middle of the leg tips. In Figure 12(b) both legs are on the support surface but the projection of the platform center is closer to the back leg tip. Figure 12(c) shows the single support of *SemiQuad*. In this Figure 12(c) the back leg is on the ground, the front leg is a swing leg. In Figure 12(d) we see the double support phase after landing of the front leg. Figure 12(e) shows the next single support of *SemiQuad*. Now the back leg is a swing leg, the front leg is on the ground. Finally we see in Figure 12(f) the next double support after landing of the back leg.

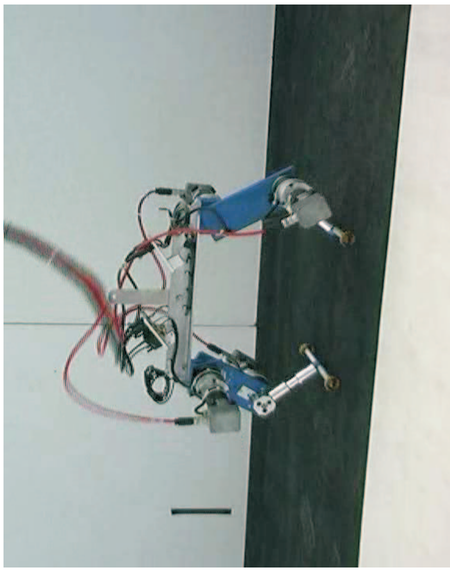
We got the last pictures from the video, and therefore at Figures 12(c), (d), (e) and (f) the vehicle image looks diffuse.

There are some differences between the experiments and the simulation. In the experiments, for example sometimes the back (front) leg looses the contact with the ground after landing of the front (back) leg. But after the back (front) leg quickly comes back to the ground and the walking continues.

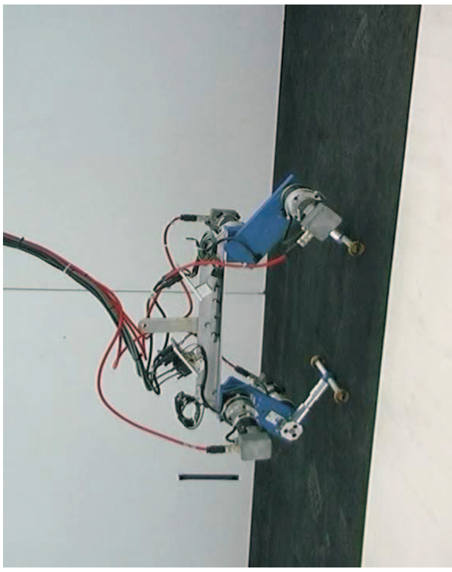
In simulation, for some parameters of *SemiQuad* and some distances between the feet at the instant of impact, we also have observed a rebound of the stance leg after an impact of the swing leg.

Multimedia material can be viewed at:
http://www.irccyn.ec-nantes.fr/irccyn/Equipes/Robotique/Themes/Mobile/images/Semi_quad.avi

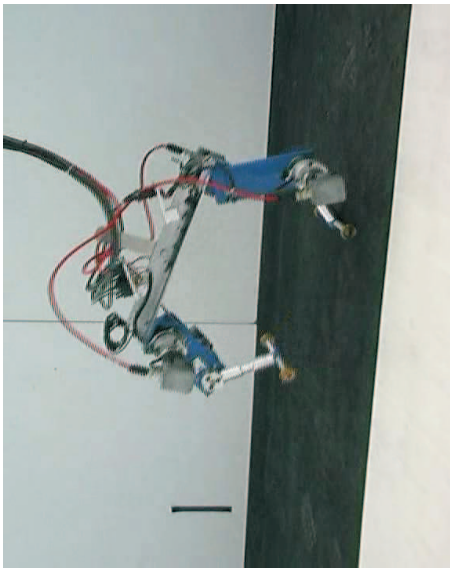
(c)



(f)

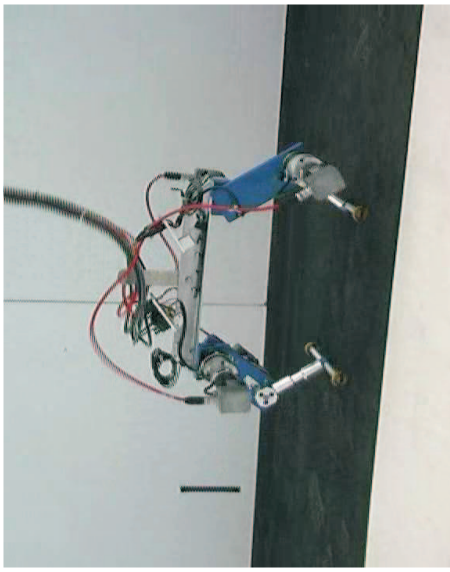


(b)



(e)

(a)



(d)

Figure 12. Walking of *SemiQuad* (from the left to the right). (a) double support of a first half step, the projection of the platform center is in the middle of the leg tips, (b) the projection of the platform center is closer to the back leg tip, (c) single support - the back leg is on the ground, the front leg is a swing leg, (d) double support after landing of the front leg, (e) single support - the front leg is on the ground, the back leg is a swing leg, (f) double support after landing of the back leg.

7. Conclusion

We present here a walking robot, which is statically stable in double support and statically unstable in single support. It is under actuated in the single support and over actuated in the double support. Its motion is planar to illustrate a *curvet* gait of a quadruped robot. We have designed an intuitive control strategy for this walking virtual quadruped - called *SemiQuad*. This robot has performed first steps. The walking of this quadruped is dynamically stable. To track the reference trajectories we use for each joint usual PD-controller. To get jumps a constant large torques (in open loop) are applied in the knees of the front or back leg.

We intend to reduce the duration of double support phase to obtain more a "dynamical" and quick walking gait.

It is possible also to obtain quicker gait by reconstructing our quadruped. With longer legs the gait can be quicker.

We are going to include the force sensors to measure and to regulate the ground reactions in the stance legs.

Appendix

Index to Multimedia Extensions: The multimedia extension page is found at:
http://www.irccyn.ec-nantes.fr/irccyn/Equipes/Robotique/Themes/Mobile/images/Semi_quad.avi

Table IV. Table of Multimedia Extension

Extension	*Type*	*Description*
1	Video	Walk of SemiQuad

Acknowledgements

The authors sincerely thank Serge Bellavoir, Gael Branchu, Damien Chablat, Philippe Lemoine and Paul Molina for their outstanding work on the experimental facilities.

References

Altendorfer, R., Moore, N., Komsuoglu, H., Buehler, M., Brown, Jr. H. B., McMordie, D., Saranli, U., Full, R., and Koditschek, D. E. 2001. RHex: A Biologically Inspired Hexapod Runner. *J. Autonoumos Robots*, 11, 207-213.

Aoustin, Y. and Formal'sky, A. M. 1998. Control Strategy for Dynamic Locomotion Design of Walking Quadruped. *Proc. of the IFAC Workshop, Motion Control 98*, pp. 401–407.

Aoustin, Y. and Formal'sky, A. M. 2003. Control design for a biped: Reference trajectory based on driven angles as functions of the undriven angle. *J. of Computer and System Sciences Int.*, 42(4), 159-176.

Chevallereau, C., Abba, G., Aoustin,Y., Plestan, F., Westervelt, A. R., Canudas de Wit,C., and Grizzle, J. W. 2003. Rabbit: A Testbed for Advanced Control Theory. *IEEE Control System Magazine*, 23(5), 57-78.

Cham, J., Bailey, S. A. and Cutkosky, M. R. 2000. Robust Dynamic Locomotion Through Feedforward-Preflex Interaction. *Proc. of the ASME Conf. Mechanical Engineers Congress and Exhibition.*

Conte, G., Scaradozzi, E., Suardi, A. 2003. Control Architecture for a Prototypal Legged Robot. *Proc. of the Clawar Conf.*, pp. 127-134.

De Lasa, M. and Buehler, M. 2001. Dynamic Compliant Quadruped Walking. *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 21-26.

Estremera J., Garcia E., Gonzalez de Santos, P. 2003. A Continuous Free Crab Gait for Quadruped Robots on Irregular Terrain. *Proc. of the Int. Conf. on Climbing and Walking Robots (CLAWAR)* , pp. 584-592.

Formal'sky, A. M., Chevallereau, C., Perrin, B. 2000. On Ballistic Walking Locomotion of a Quadruped. *Int. J. of Robotic Research*, 19(8), 743-761.

Formal'sky, A. M. 1982. *Locomotion of Anthropomorphic Mechanisms*. *Nauka*, Moscow. In Russian

Formal'sky, A. M. 1997. Ballistic Locomotion of a Biped. Design and Control of Two Biped Machines. In: Human and Machine Locomotion, ed. A.Morecki and K.Waldron. *Udine, Italy: CISM, Springer-Verlag*.

Fukuoka, Y., Kimura, H., Cohen, A. H. 2003. Adaptative Dynamic Walking of a Quadruped Robot on Irregular Terrain Based on Biological Concepts. *Int. J. of Robotic Research*, 22(3-4), 187-202.

Furusho, J., Sano, A., Sakaguchi, M. and Koizumi, E. 1995. Realization of Bounce Gait in a Quadruped Robot with Articular-Joint-Type Legs. *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 697–702.

Gurfinkel, V. S, Gurfinkel, E. V, Shneider, A. Y, Devjanin, E. A, Lensky, A. V. and Shtilman, L. G. 1981. Walking Robot with Supervisory Control. *Mechanisms and Machines Theory*, 16(2), 31-36.

Hirai, K., Hirose, M., Haikawa, Y., and Takenaka, T. 1998. The Development of Honda Humanod Robot. *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 1321–1326.

Hirose, S. and Kato, K. 2000. Study on Quadruped Walking Robot in Tokyo Institute of Technology. *Proc. of the IEEE Conf. on Robotics and Automation*, pp. 414-419.

Hirose, S. and Yoneda, K. 1993. Toward Development of Practical Quadruped Walking Vehicles. *J. of Robotics and Mechatronics*, 5(6), 498-504.

Hong, Y. S., Lee, H. K, Yi, S. Y and Lee, C. W. 1999. The Design and Control of a Jointed-Leg Type of a Quadrupedal Robot for Locomotion on Irregular Ground. *Robotica*, 17(4), 383–389.

Kaneko, M., Abe, M. and Tanie, K. 1985. A Hexapod Walking Machine with Decoupled Freedoms. *IEEE J. of Robotics and Automation*, 1(4), 183–190.

Kimura, H., Akiyama, S., Sakurama, K. 1999. Realization of Dynamic and Running of the Quadruped using Neural Oscillator. *J. Autonoumos Robots*, 7, 247-258.

Klein, C. A. and Briggs, R. L. 2000. Use of Active Compliance in the Control of Legged Vehicles. *IEEE Transactions on Systems, Man and Cybernetics*, 10(7), 393-400.

Krupp B. T. 2003. Preliminary Control of a Planar Robot for Quadrupedal Locomotion Research. *Proc. of the Int. Conf. on Climbing and Walking Robots (CLAWAR)*, pp. 95-104.

Miura, H. and Shimoyama, I. 1984. Dynamic Walk of a Biped. *Int. J. of Robotics Research*, 3(2), 60–74.

Muraro, A. Chevallereau, C. and Aoustin, Y. 2003. Optimal Trajectories for a Quadruped Robot with *Trot*, *Amble*, and *Curvet* Gaits for two Energetic Criteria. *Int. J. of Multibody System Dynamics*, 9, 39–62.

Pratt, G. A. and Williamson, M. M. 1995. Series Elastic Actuators. *Proc. of the IEEE Conf. on Intelligent Robots and Systems*, pp. 399–406.

Poulakakis, I. Smith, J. A and Buehler, M. 2003. On the Dynamics of Bounding and Extensions Torwards the Half-Bound and the Gallop Gaits. *Proc. of the Int. Symp. on Adaptative Motion of Animals and Machines, Kyoto.*

Perrin, B. 1999. Modelling and Control of a Quadruped for a Dynamically Stable Gait. *Ph D Thesis, University of Nantes, in french.*

Saranli, U., Buehler, M. and Koditschek, D. E. 2001. Rhex: A Simple and Highly Mobile Hexapod Robot. *Int. J. of Robotic Research*, 20(7):616-631.

Sutherland, I. E. and Raibert, M. H. 1983. Machines that Walk. *Scientific American*, 248:44-53.

Talebi S., Poulakakis G., Papadopoulos, E. and Buehler, M. 2001. Quadruped Robot Running with a Bounding Gait. Daniela Rus and Sanjiv Singh, editors, Experimental Robotics VII, in. *Lecture Notes in Control and Information Sciences*, 271:281-289.

Ting S. H., Blickhan R. and Full, R. F. 1994. Dynamic and Static Stability in Hexapedal Runners. *J. of Experimental Biology*, 197:251-269.

Vukobratovic, M., Borovac, B., Surla, D. and Stokic, D. 1990. *Biped Locomotion*. *Springer-Verlag*, 349 p.